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Qualcomm

Dead Iteration Elimination

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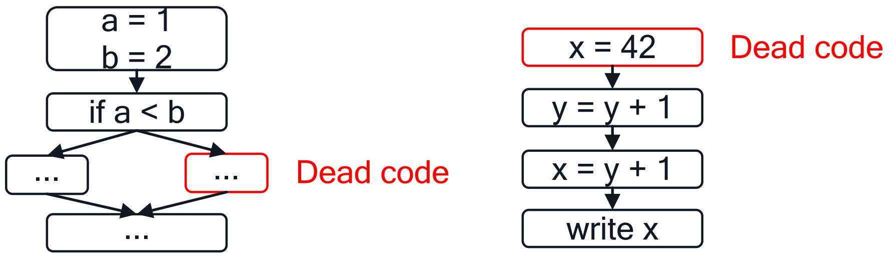
We are not monsters

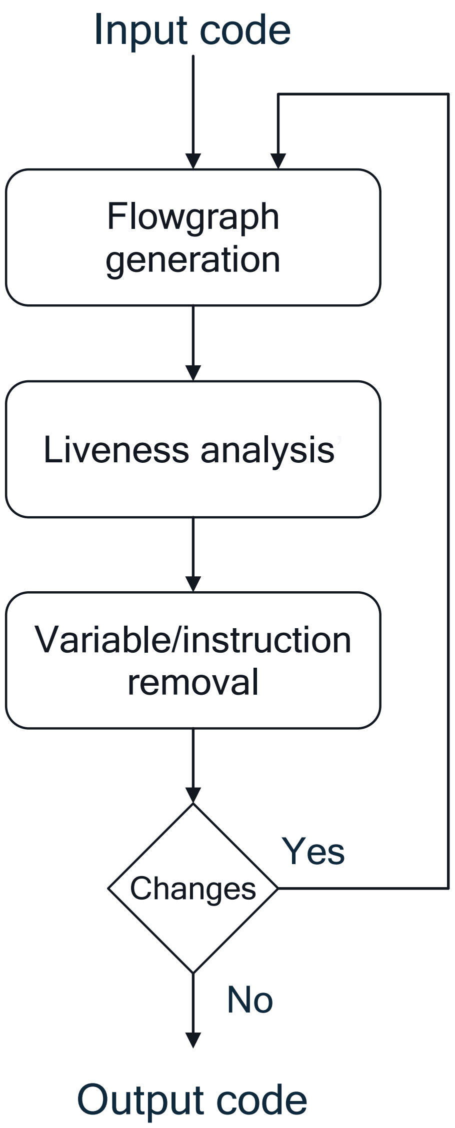
No iterations were harmed during the making of this presentation!

Prior Art

- **Dead Code Elimination (DCE)**
  - Elimination of code either never executed or producing never used data

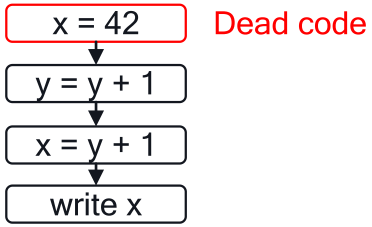


  - Exploit SSA form and data-flow analysis information [Cytron et al. TOPLAS 91]
  - Simple analysis of the uses of values and reachability of instructions
  - Iterative elimination until stability
- **Target complete removal of instructions**
  - Avoid allocating memory for unused variables
  - May ultimately remove full functions
  - No consideration of loop iteration spaces and removal of dead iterations

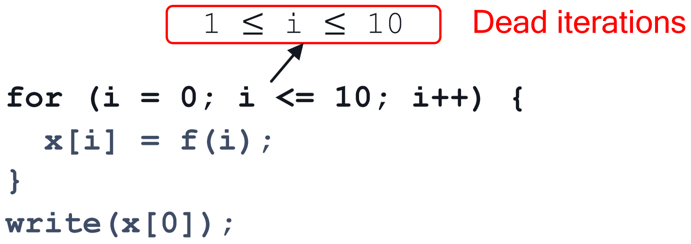


Introduction

- **Prior art: Dead Code Elimination (DCE)**
  - Classical optimization present in all compilers
  - Identification and elimination of <u>code</u> not contributing to output data
  - Increase performance and reduce executable size
  - Target complete removal of instructions



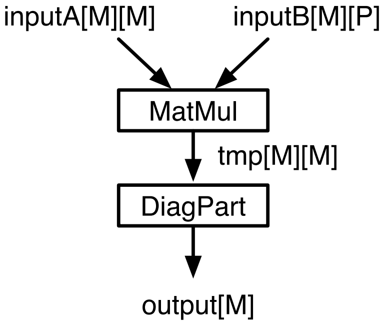
- **Our Approach: Dead Iteration Elimination (DIE)**
  - Identification and elimination of <u>loop iterations</u> not contributing to output data
  - Relevant to graph compilers compiling applications built from standard “bricks”
    - Operator composition
    - Inactivation of iterations due to, e.g., specialization/sparsification/subsampling
  - Complementary to DCE



Application Scenarios 1/4

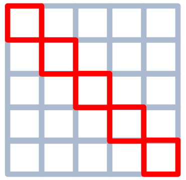
Iteration-level dead code elimination

- Remove iterations not contributing to the output (because of operator fusion, or bad programming)
- Example: MatMul-DiagPart subgraph filtering diagonal elements after a matrix multiplication




```
// MatMul operator
for (i = 0; i < M; i++) {
  for (j = 0; j < M; j++) {
    tmp[i][j] = 0.;
    for (k = 0; k < P; k++) {
      tmp[i][j] += inputA[i][k] * inputB[k][j];
    }
  }
}
// DiagPart operator
for (i = 0; i < M; i++) {
  output[i] = tmp[i][i];
}
```


```
// Fused MatMul-DiagPart operator after DIE
for (i = 0; i < M; i++) {
  temp[i][i] = 0.;
  for (k = 0; k < P; k++) {
    temp[i][i] += inputA[i][k] * inputB[k][i];
  }
}
for (i = 0; i < M; i++) {
  output[i] = temp[i][i];
}
```



Computation not necessary to the diagonal output has been filtered out

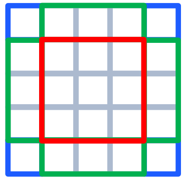
- Reduce the need for specialized custom operators in AI/DL frameworks

Application Scenarios 2/4

Tile-specialized code generation

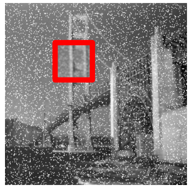
- Given a set of output tiles and a general code, generate the code computing only that set of output tiles
- Example: general mean filter specialized to a given tile

```
for (i = 0; i < height; i++) {
  for (j = 0; j < width; j++) {
    if ((i == 0) && (j == 0)) {                                                  // Case 1: top left corner
      output[i][j] = (input[i][j] + input[i][j+1] + input[i+1][j] + input[i+1][j+1]) / 4;
    }
    if ((i == 0) && (j > 0) && (j < width - 1)) {                                // Case 2: top row except corners
      output[i][j] = (input[i][j-1] + input[i][j] + input[i][j+1] + input[i+1][j-1] + input[i+1][j] + input[i+1][j+1]) / 6;
    }
    if ((i == 0) && (j == width - 1)) {                                          // Case 3: top right corner
      output[i][j] = (input[i][j-1] + input[i][j] + input[i+1][j-1] + input[i+1][j]) / 4;
    }
    if ((i > 0) && (i < height - 1) && (j == 0)) {                               // Case 4: left column except corners
      output[i][j] = (input[i-1][j] + input[i-1][j+1] + input[i][j] + input[i][j+1] + input[i+1][j] + input[i+1][j+1]) / 6;
    }
    if ((i > 0) && (i < height - 1) && (j > 0) && (j < width - 1)) {             // Case 5: out of borders, general case
      output[i][j] = (input[i-1][j-1] + input[i-1][j] + input[i-1][j+1] +
                      input[i  ][j-1] + input[i  ][j] + input[i  ][j+1] +
                      input[i+1][j-1] + input[i+1][j] + input[i+1][j+1]) / 9;
    }
    if ((i > 0) && (i < height - 1) && (j == width - 1)) {                       // Case 6: right column except corners
      output[i][j] = (input[i-1][j-1] + input[i-1][j] + input[i][j-1] + input[i][j] + input[i+1][j-1] + input[i+1][j]) / 6;
    }
    if ((i == height - 1) && (j == 0)) {                                         // Case 7: bottom left corner
      output[i][j] = (input[i-1][j] + input[i-1][j+1] + input[i][j] + input[i][j+1]) / 4;
    }
    if ((i == height - 1) && (j > 0) && (j < width - 1)) {                       // Case 8: bottom row except corners
      output[i][j] = (input[i-1][j-1] + input[i-1][j] + input[i-1][j+1] + input[i][j-1] + input[i][j] + input[i][j+1]) / 6;
    }
    if ((i == height - 1) && (j == width - 1)) {                                 // Case 9: bottom right corner
      output[i][j] = (input[i-1][j-1] + input[i-1][j] + input[i][j-1] + input[i][j]) / 4;
    }
  }
}
```




```
for (i = 64; i < 128; i++) {
  for (j = 64; j < 128; j++) {
    output[i][j] = (input[i-1][j-1] + input[i-1][j] + input[i-1][j+1] +
                    input[i  ][j-1] + input[i  ][j] + input[i  ][j+1] +
                    input[i+1][j-1] + input[i+1][j] + input[i+1][j+1]) / 9;
  }
}
```

Code specialized to the desired tile output[64..127][67..127] only



Application Scenarios 3/4

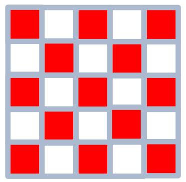
Sparsification/subsampling

- Given a dense operator and a structured sparsity information, generate the code producing only the desired data
- Example: checkerboard subsampling

```
// Roberts Edge Detection Filter
for (i = 0; i < height - 3; i++) {
  for (j = 0; j < width - 3; j++) {
    output[i+1][j+2] = abs(tmp1[i+1][j+2] - tmp1[i+2][j+1]) +
                       abs(tmp1[i+2][j+2] - tmp1[i+1][j+1]);
  }
}
```

DIE

```
for (i = 0; i < height - 3; i++) {
  for (j = 0; j < width - 3; j++) {
    if ((i + j - 3) % 2 == 0) {
      output[i+1][j+2] = abs(tmp1[i+1][j+2] - tmp1[i+2][j+1]) +
                         abs(tmp1[i+2][j+2] - tmp1[i+1][j+1]);
    }
  }
}
```



Code updated to compute only the desired data
(here, checkerboard-style: output[d1][d2] with (d1+d2)%2 == 0)

Application Scenarios 4/4

Compiler warning

- Compiler warns the user that some iterations are (missing or) not contributing to the data space
- Example: out of bound access that SOTA static checkers (e.g., cppchecker or Clang's scan-build) fail to find

```
void init(size_t size, int vector[size]) {
  for (size_t i = 0; i < size; i++) {
    vector[i + 1] = 0;
  }
}
```

DIE

```
  for (size_t i = 0; i < size; i++) {
  ^
out.c:4:3: iteration "i <- size" leads to out of bound access of vector[i + 1]
```

Non-trivial out of bound access has been detected at compilation time

Problem Statement

- **Problem: programs may include statement iterations not contributing to the desired output**
  - **Graph compilers**
    - Excess computation may be introduced by operator sequences (e.g., when some output of a given operator is not used)
    - Iteration domains of basic operators may be reduced due to specialization, sparsification, subsampling
  - **General compilers**
    - Parameters may remove some computation or output while corresponding computation (partly) remain
    - Bugs may be present in the code

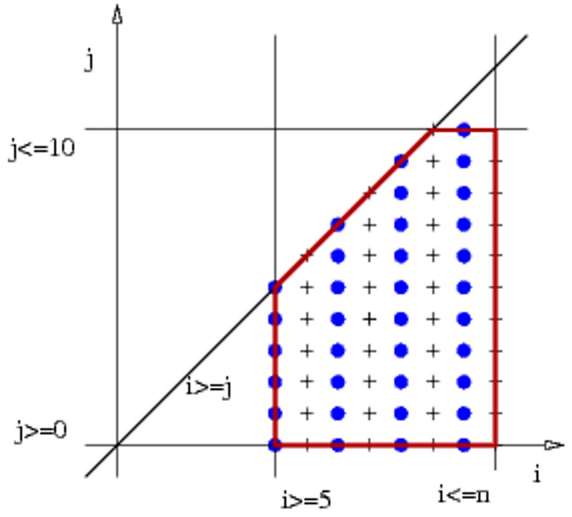
- **Relevance: applications to optimization and productivity**
  - **For computational graph compilers**
    - Enable automatic removal of excess computation not contributing to the final output, reduce the need for custom operators
    - Enable automatic specialization, sparsification and subsampling of (sequences of) operators, including custom operators
  - **For general compilers**
    - Complement dead-code elimination with dead iteration elimination
    - Extend static analysis ability to detect potential bugs, e.g., out of bound memory accesses

Solution: Polyhedral Model Background

- **Models the execution of loop nests in a vector space**
  - Loop iterations and data accesses are integer-valued points within a polyhedron
  - Analyses and transformations rely on high-level math libraries
    - ➢E.g., our algorithm relies on fusion, intersection, projection, image, preimage of polyhedra
  - Algorithms exist for many analyses and tasks
    - ➢E.g., our algorithm relies on exact iteration-wise data dependence analysis
  - Code or compiler IR can be generated from the polyhedral representation

- **A vast majority of Neural Net layers can be efficiently modeled using the polyhedral model [TACO 2019]**
  - But the technology has a broader application domain (including radar, image & signal processing, physics simulations)

```
n = f();
for (i = 5; i <= n; i += 2) {
  A[i][i] = A[i][i] / B[i];
  for (j = 0; j <= i; j++) {
    if (j <= 10) {
      ... A[i + 2*j + n][i + 3] ...
    }
  }
}
```

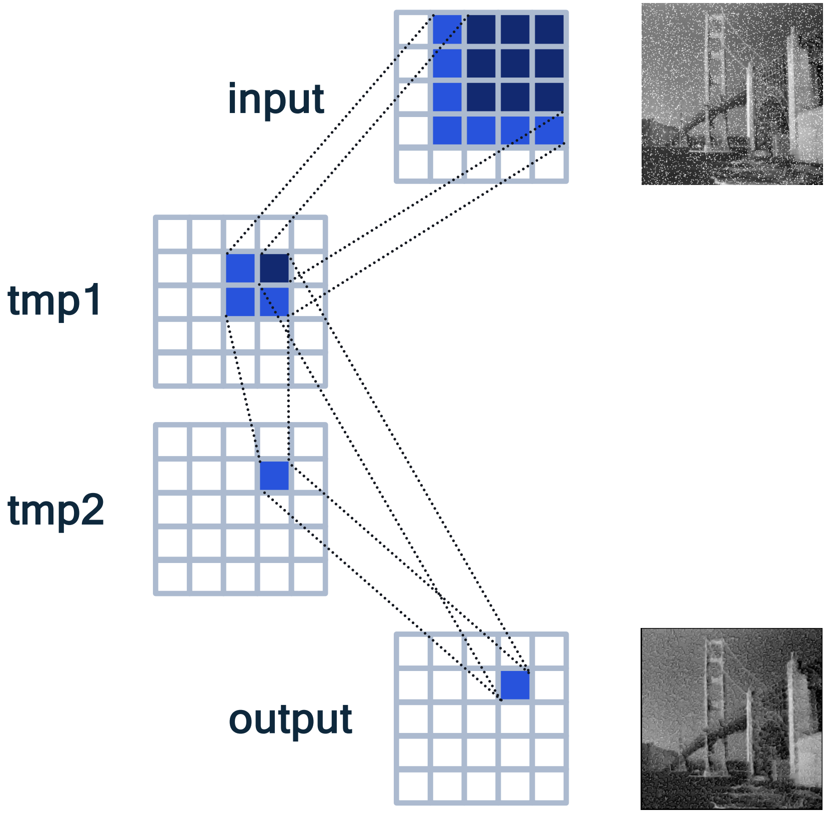


$$\{i, j \in \mathbb{Z}^2 | \exists k \in \mathbb{Z}, \\ 5 \leq i \leq n; 0 \leq j \leq i; i = 2k + 1\}$$

Solution: Introductive Example

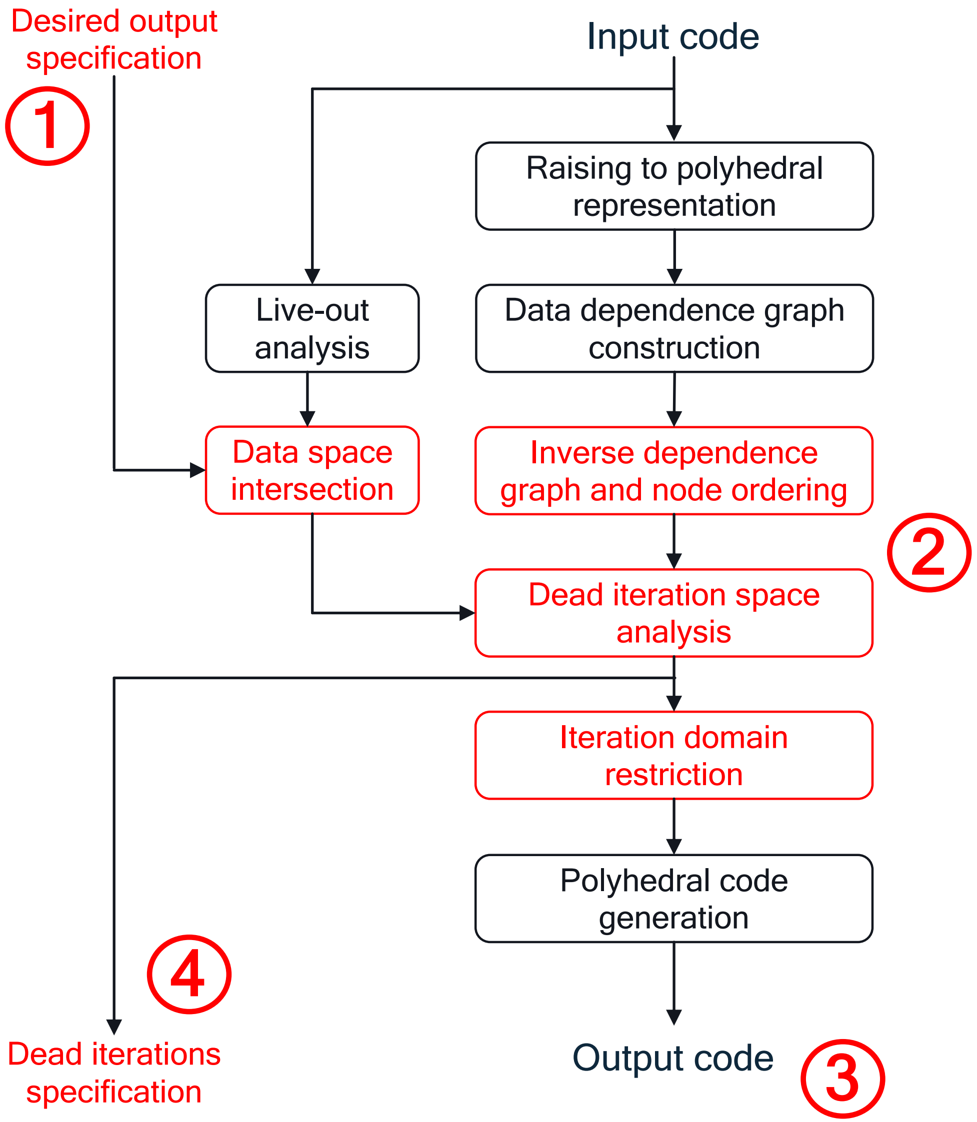
- Mean-Roberts Sharpening
  - 3-stage image processing pipeline
- To produce a given part of the final image, which data and computations are required?

```
// Mean Filter
for (i = 1; i < height - 1; i++) {
  for (j = 1; j < width - 1; j++) {
    tmp1[i][j] = (input[i-1][j-1] + input[i-1][j] + input[i-1][j+1] +
                  input[i  ][j-1] + input[i  ][j] + input[i  ][j+1] +
                  input[i+1][j-1] + input[i+1][j] + input[i+1][j+1]) / 9;
  }
}
// Roberts Edge Detection Filter
for (i = 0; i < height - 3; i++) {
  for (j = 0; j < width - 3; j++) {
    tmp2[i+1][j+2] = abs(tmp1[i+1][j+2] - tmp1[i+2][j+1]) +
                     abs(tmp1[i+2][j+2] - tmp1[i+1][j+1]);
  }
}
// Additive
for (i = 1; i < height - 2; i++) {
  for (j = 2; j < width - 1; j++) {
    output[i][j] = tmp1[i][j] - 1 * tmp2[i][j];
  }
}
```



Solution: Key Points

1. **Support for desired output data specification**
   - Enable specialization/sparsification/subsampling specification
2. **Iteration-level dead code analysis**
   - Innovative polyhedral analysis dedicated to required data-space analysis and their mapping to statement iteration domains
3. **Generation of output code cleared from dead iterations**
   - Exploit polyhedral code generation on restricted iteration domains
4. **Support for dead iteration space output**
   - Enable programmer feedback, e.g., compiler warnings



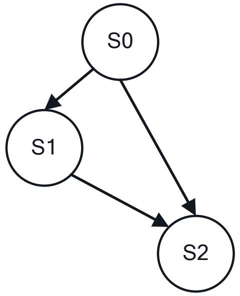
Solution: Dead Iteration Space Analysis

Principle: back-propagate data shape constraints through operations on these shapes and update iteration domains accordingly

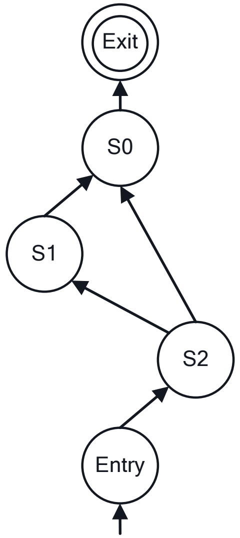
1. Build the inverted data dependence graph (inverted edges), including:
   - A new entry vertex with edges to every vertex writing the output tensors
   - A new exit vertex with edges from every vertex reading the input tensors
   - Nodes involved within a dependence cycle merged into a supernode
2. For each vertex v, initialize $R_{v,t}$ the required data space for the tensor t:
   - Entry vertex: $R_{entry,t}$ = required data space of each output tensor t
   - Other vertices: empty spaces
3. For each vertex v according to a topological ordering of the nodes (v has iteration domain D_v, writes tensor w with function f_w, and reads tensors r_i with function f_{ri})
   a. Vertex potential contribution space for the written tensor w: $P_{v,w} = \bigcup_{i \text{ in predecessor}(v)} R_{i,w}$
   b. Vertex contributing space for the written tensor w: $C_{v,w} = \text{Preimage}(P_{v,w}, f_w) \cap D_v$
   c. Required data spaces for v: $\{R_{v,ri} = \text{Image}(C_{v,w}, f_{ri})\}$
   d. Dead iteration space for v: $\text{Dead}_v = D_v - C_{v,w}$
4. For each vertex v, restrict the iteration domain:
   - $D_v = D_v - \text{Dead}_v$

```
// Mean Filter
for (i = 1; i < height - 1; i++) {
  for (j = 1; j < width - 1; j++) {
S0: tmp1[i][j] = (input[i-1][j-1] + input[i-1][j] + input[i-1][j+1] +
                  input[i  ][j-1] + input[i  ][j] + input[i  ][j+1] +
                  input[i+1][j-1] + input[i+1][j] + input[i+1][j+1]) / 9;
  }
}
// Roberts Edge Detection Filter
for (i = 0; i < height - 3; i++) {
  for (j = 0; j < width - 3; j++) {
S1: tmp2[i+1][j+2] = abs(tmp1[i+1][j+2] - tmp1[i+2][j+1]) +
                     abs(tmp1[i+2][j+2] - tmp1[i+1][j+1]);
  }
}
// Additive
for (i = 1; i < height - 2; i++) {
  for (j = 2; j < width - 1; j++) {
S2: output[i][j] = tmp1[i][j] - 1 * tmp2[i][j];
  }
}
```

Data-Dependence Graph



Inverted Dependence Graph With Entry/Exit Nodes

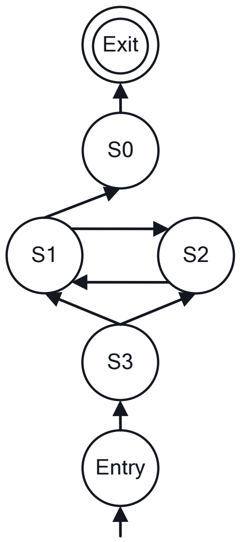


Required data space: output[LB0..UB0][LB1..UB1]
$R_{entry,output} = \{(d0,d1) \mid LB0 \leq d0 \leq UB0, LB1 \leq d1 \leq UB1\}$

Cycle Handling

- Time constrained (production compiler)
  - Use “lightweight” (complexity) set operations such as image/preimage/union/subtract
  - Only visit node once!
- Locate the strongly connected components
  - Over-approximate by requiring the node’s full iteration domain to be visited!
- Simple but very effective accuracy improvement
  - Don’t approximate self dependency cycle with distance zero
  - E.g., reduction X += A[i], or more generally X = f(X, …)

```
for (i = 0; i <= 10; i++) {
  tmp1[i] = input[i];             // S0
}
for (i = 5; i <= 10; i++) {
  tmp2[i] = tmp1[i] + tmp3[i - 5]; // S1
  tmp3[i - 4] += tmp2[i - 2];      // S2
}
for (i = 0; i <= 10; i++) {
  output[i] = tmp2[i] + tmp3[i];   // S3
}
```



Example:

- In this case with a dependence cycle, only the ordering Entry → S3 → S2 → S1 → S0 → Exit is valid, not Entry → S3 → S1 → S2 → S0 → Exit (because there is no edge to propagate information coming from S2 to S0).

Polyhedral Art

Sven Verdoolaege proposed a variant of the algorithm

- Implemented in PPCG and leveraged by Polly (LLVM) and AlphaZ

Fixed point algorithm starting from the live-out set:

1. **Apply reversed dependence map**

   Stop if no additional iterations have been added

2. **Union w/ the previous iteration set**

3. **Widening using affine hull**

4. **Intersect w/ the Scop domain**

Differences w/ Sven's Algorithm

Widening for depencency cycle handling

Most implementation of Sven's algorithm use affine hull as widening operation:

- Rather expensive to compute
- Too coarse? (at Scop level)

Our algorithm:

- Lighter; one traversal of the inverted dependance graph, one node at a time (not full scop)
- Dependence cycles are approximated (apart for self dependency)

Matmul w/ triangular output

```
for (i = 0; i < M; i++)
  for (j = 0; j < M; j++) {
    tmp[i][j] = 0.;
    for (k = 0; k < P; k++)
      tmp[i][j] += inputA[i][k] * inputB[k][j];
  }
for (i = 0; i < M; i++)
  for (j = i ; j < M; j++)
    output[i][j] = tmp[i][j];
```

```
for (i = 0; i < M; i++)
  for (j = i; j < M; j++) {
    tmp[i][j] = 0.;
    for (k = 0; k < P; k++)
      tmp[i][j] += inputA[i][k] * inputB[k][j];
  }
for (i = 0; i < M; i++)
  for (j = i; j < M; j++)
    output[i][j] = tmp[i][j];
```

Summary

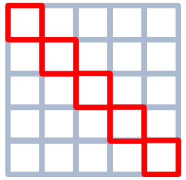
- Extracts the **iteration** and **data domains** required given an output data specification
  - For each node of the computation graph
- Applicable to a wide variety of program
  - Static Control Parts
  - + black box operations where the iteration domain and access function are approximated
- Many application scenarios
  - Code analysis and feedback/warning generation
  - Remove dead iterations from existing code base (e.g., due to fusion, composition, etc)
  - Generate code tailored to computing a subset of an original output space (tilingspecialization, etc)

Application Scenarios 1/2

- **Iteration-level dead code elimination**
  - Remove the iterations not contributing to the output (because of operator fusion, or bad programming)
  - Example: filtering out diagonal elements after a matrix multiplication

```
for (i = 0; i < M; i++) {   // Matrix multiplication kernel
  for (j = 0; j < M; j++) {
    temp[i][j] = 0.;
    for (k = 0; k < P; k++) {
      temp[i][j] += inputA[i][k] * inputB[k][j];
    }
  }
}
for (i = 0; i < M; i++) {   // Filtering of diagonal elements
  output[i] = temp[i][i];
}
```

```
// NOTE: additional analysis may entirely remove temp array
for (i = 0; i < M; i++) {
  temp[i][i] = 0.;
  for (k = 0; k < P; k++) {
    temp[i][i] += inputA[i][k] * inputB[k][i];
  }
  output[i] = temp[i][i];
}
```

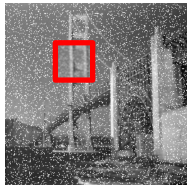


Computation not necessary to compute the diagonal has been filtered out

- **Tile-specialized code generation**
  - Given a set of output tiles and a general code, generate the code that computes only that set of output tiles
  - Example: general mean filter specialized to a given tile

```
for (i = 0; i < height; i++) {
  for (j = 0; j < width; j++) {
    if ((i == 0) && (j == 0)) {                                            // Case 1: top left corner
      output[i][j] = (input[i][j] + input[i][j+1] + input[i+1][j] + input[i+1][j+1]) / 4;
    }
    if ((i == 0) && (j > 0) && (j < width - 1)) {                          // Case 2: top row except corners
      output[i][j] = (input[i][j-1] + input[i][j] + input[i][j+1] + input[i+1][j-1] + input[i+1][j] + input[i+1][j+1]) / 6;
    }
    if ((i == 0) && (j == width - 1)) {                                    // Case 3: top right corner
      output[i][j] = (input[i][j-1] + input[i][j] + input[i+1][j-1] + input[i+1][j]) / 4;
    }
    if ((i > 0) && (i < height - 1) && (j == 0)) {                         // Case 4: left column except corners
      output[i][j] = (input[i-1][j] + input[i-1][j+1] + input[i][j] + input[i][j+1] + input[i+1][j] + input[i+1][j+1]) / 6;
    }
    if ((i > 0) && (i < height - 1) && (j > 0) && (j < width - 1)) {       // Case 5: out of borders, general case
      output[i][j] = (input[i-1][j-1] + input[i-1][j] + input[i-1][j+1] +
                      input[i  ][j-1] + input[i  ][j] + input[i  ][j+1] +
                      input[i+1][j-1] + input[i+1][j] + input[i+1][j+1]) / 9;
    }
    if ((i > 0) && (i < height - 1) && (j == width - 1)) {                 // Case 6: right column except corners
      output[i][j] = (input[i-1][j-1] + input[i-1][j] + input[i][j-1] + input[i][j] + input[i+1][j-1] + input[i+1][j]) / 6;
    }
    if ((i == height - 1) && (j == 0)) {                                   // Case 7: bottom left corner
      output[i][j] = (input[i-1][j] + input[i-1][j+1] + input[i][j] + input[i][j+1]) / 4;
    }
    if ((i == height - 1) && (j > 0) && (j < width - 1)) {                 // Case 8: bottom row except corners
      output[i][j] = (input[i-1][j-1] + input[i-1][j] + input[i-1][j+1] + input[i][j-1] + input[i][j] + input[i][j+1]) / 6;
    }
    if ((i == height - 1) && (j == width - 1)) {                           // Case 9: bottom right corner
      output[i][j] = (input[i-1][j-1] + input[i-1][j] + input[i][j-1] + input[i][j]) / 4;
    }
  }
}
```

```
for (i = 64; i < 128; i++) {
  for (j = 64; j < 128; j++) {
    output[i][j] = (input[i-1][j-1] + input[i-1][j] + input[i-1][j+1] +
                    input[i  ][j-1] + input[i  ][j] + input[i  ][j+1] +
                    input[i+1][j-1] + input[i+1][j] + input[i+1][j+1]) / 9;
  }
}
```



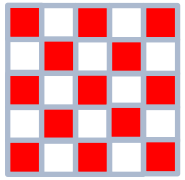
Code has been specialized to the desired tile only

Application Scenarios 2/2

- **Sparsification/subsampling**
  - Given a dense operator and a structured sparsity information, generate the code producing only the desired data
  - Example: checkerboard subsampling

```c
// Roberts Edge Detection Filter
for (i = 0; i < height - 3; i++) {
  for (j = 0; j < width - 3; j++) {
    output[i+1][j+2] = abs(tmp1[i+1][j+2] - tmp1[i+2][j+1]) +
                       abs(tmp1[i+2][j+2] - tmp1[i+1][j+1]);
  }
}
```

```c
// NOTE: additional analysis may output on subsampled matrix
for (i = 0; i < height - 3; i++) {
  for (j = 0; j < width - 3; j++) {
    if ((i + j - 3) % 2 == 0) {
      output[i+1][j+2] = abs(tmp1[i+1][j+2] - tmp1[i+2][j+1]) +
                         abs(tmp1[i+2][j+2] - tmp1[i+1][j+1]);
    }
  }
}
```



Code updated to compute only the desired data (here, checkerboard-style)

- **Compiler warning**
  - Compiler warns the user that some iterations are (missing or) not contributing to the data space
  - Example: out of bound access that advanced static checkers (e.g., cppchecker or Clang’s scan-build) are failing to find

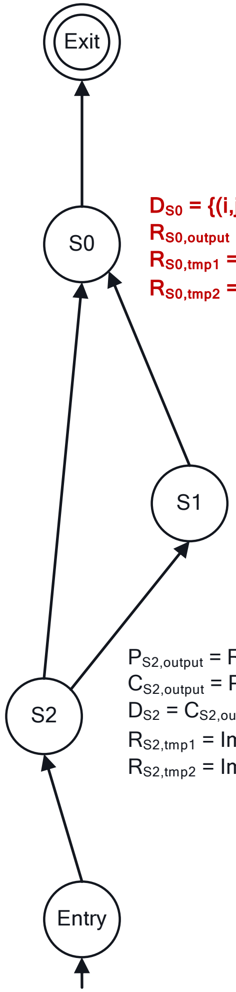
```c
void init(size_t size, int vector[size]) {
  for (size_t i = 0; i < size; i++) {
    vector[i + 1] = 0;
  }
}
```

```
  for (size_t i = 0; i < size; i++) {
  ^
out.c:4:3: iteration "i <- size" leads to out of bound access of vector[i + 1]
```

Non-trivial out of bound access has been detected at compilation time

DIE General Algorithm

Inverted Dependence Graph
With Entry/Exit Nodes



$D_{S0} = \{(i,j) \mid 1 \le i \le height, 1 \le j \le width\}$
$R_{S0,output} = \emptyset$
$R_{S0,tmp1} = \emptyset$
$R_{S0,tmp2} = \emptyset$

$P_{S1,tmp2} = R_{S2,tmp2} = \{(d_0, d_1, i_2, j_2) \mid d_0 = i_2 \;\&\&\; d_1 = j_2 \;\&\&\; 1,LB0 \le i_2 \le height - 1,UB0 \;\&\&\; 2, LB1 \le j_2 \le width, UB1\}$
$C_{S1,tmp2} = Preimage(P_{S1,tmp2}, (d_0 = i_1 + 1, d_1 = j_1 + 2)) \cap D_{S1} = \{(i_1, i_2, j_1, j_2) \mid i_1 = i_2 \;\&\&\; j_1 = j_2 \;\&\&\; 1,LB0 \le i_2 \le height - 1,UB0 \;\&\&\; 2, LB1 \le j_2 \le width, UB1\} \cap \{(i,j) \mid 0 \le i \le height - 2, 0 \le j \le width - 2\}$
$= \{(i_1, i_2, j_1, j_2) \mid i_1 = i_2 \;\&\&\; j_1 = j_2 \;\&\&\; 1,LB0 \le i_2 \le height - 2,UB0 \;\&\&\; 2, LB1 \le j_2 \le width - 2, UB1\}$
$D_{S2} = C_{S2,output} = \{(i_2, j_2) \mid 1,LB0 \le i_2 \le height - 1,UB0 \;\&\&\; 2, LB1 \le j_2 \le width, UB1\}$
$R_{S2,tmp1} = Image(C_{S2,output}, (d_0 = i_2, d_1 = j_2)) = \{(d_0, d_1, i_2, j_2) \mid d_0 = i_2 \;\&\&\; d_1 = j_2 \;\&\&\; 1,LB0 \le i_2 \le height - 1,UB0 \;\&\&\; 2, LB1 \le j_2 \le width, UB1\}$
$R_{S2,tmp2} = Image(C_{S2,output}, (d_0 = i_2, d_1 = j_2)) = \{(d_0, d_1, i_2, j_2) \mid d_0 = i_2 \;\&\&\; d_1 = j_2 \;\&\&\; 1,LB0 \le i_2 \le height - 1,UB0 \;\&\&\; 2, LB1 \le j_2 \le width, UB1\}$

$D_{S1} = \{(i,j) \mid 0 \le i \le height - 2, 0 \le j \le width - 2\}$
$R_{S1,output} = \emptyset$
$R_{S2,tmp1} = \emptyset$
$R_{S1,tmp2} = \emptyset$

$P_{S2,output} = R_{entry,output} = \{(d_0, d_1) \mid LB0 \le d_0 \le UB0 \;\&\&\; LB1 \le d_1 \le UB1\}$
$C_{S2,output} = Preimage(P_{S2,output}, (d_0 = i_2, d_1 = j_2)) \cap D_{S2} = \{(i_2, j_2) \mid LB0 \le i_2 \le UB0 \;\&\&\; LB1 \le j_2 \le UB1\} \cap \{(i_2,j_2) \mid 1 \le i_2 \le height - 1 \;\&\&\; 2 \le j_2 \le width\} = \{(i_2, j_2) \mid 1,LB0 \le i_2 \le height - 1,UB0 \;\&\&\; 2, LB1 \le j_2 \le width, UB1\}$
$D_{S2} = C_{S2,output} = \{(i_2, j_2) \mid 1,LB0 \le i_2 \le height - 1,UB0 \;\&\&\; 2, LB1 \le j_2 \le width, UB1\}$
$R_{S2,tmp1} = Image(C_{S2,output}, (d_0 = i_2, d_1 = j_2)) = \{(d_0, d_1, i_2, j_2) \mid d_0 = i_2 \;\&\&\; d_1 = j_2 \;\&\&\; 1,LB0 \le i_2 \le height - 1,UB0 \;\&\&\; 2, LB1 \le j_2 \le width, UB1\}$
$R_{S2,tmp2} = Image(C_{S2,output}, (d_0 = i_2, d_1 = j_2)) = \{(d_0, d_1, i_2, j_2) \mid d_0 = i_2 \;\&\&\; d_1 = j_2 \;\&\&\; 1,LB0 \le i_2 \le height - 1,UB0 \;\&\&\; 2, LB1 \le j_2 \le width, UB1\}$

Required data space: output[LB0..UB0][LB1..UB1]
$R_{entry,output} = \{(d0,d1) \mid LB0 \le d0 \le UB0, LB1 \le d1 \le UB1\}$

```
// Mean Filter
for (i = 1; i < height - 1; i++) {
  for (j = 1; j < width - 1; j++) {
S0: tmp1[i][j] = (input[i-1][j-1] + input[i-1][j] + input[i-1][j+1] +
                  input[i  ][j-1] + input[i  ][j] + input[i  ][j+1] +
                  input[i+1][j-1] + input[i+1][j] + input[i+1][j+1]) / 9;
  }
}
// Roberts Edge Detection Filter
for (i = 0; i < height - 3; i++) {
  for (j = 0; j < width - 3; j++) {
S1: tmp2[i+1][j+2] = abs(tmp1[i+1][j+2] - tmp1[i+2][j+1]) +
                     abs(tmp1[i+2][j+2] - tmp1[i+1][j+1]);
  }
}
// Additive
for (i = 1; i < height - 2; i++) {
  for (j = 2; j < width - 1; j++) {
S2: output[i][j] = tmp1[i][j] - 1 * tmp2[i][j];
  }
}
```

Problem Statement

Remove statement iterations not contributing to a computation output

- **Provide analyses and removal techniques to identify and eliminate non-useful statement iterations from a program**
  - Input is a computational kernel or function code
  - Output is the identification of non-useful statement iterations and/or a semantically equivalent code cleared from those iterations
- **Definitions**
  - Statement iterations:
    - Subset of the executions of a statement enclosed within an iterative loop
    - Contributing statement iterations are those writing to a memory location which is part of the output
  - Computation output:
    - Live-out data space of a computational kernel or a function as analyzed by a compiler
    - Or, output-data space (e.g., output tile, sparsified/subsampled output) as specified by the user
- **Application domain**
  - Ideal application domain is known as “static control codes”
    - Loop bounds and conditionals are affine expressions of outer loop counters and constant parameters
    - Tensor access functions are affine expressions of outer loop counters and constant parameters
  - Not respecting the above conditions is not blocking but may limit non-contributing statement iteration removal
  - Typical target codes are AI/DL operators or scientific computation kernels