Polyhedral Scheduling and Relaxation of Synchronous Reactive Systems

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INRIA, Google, ENS, Airbus, Safran

June 20th, 2022

Application considered

Critical embedded real-time synchronous applications (ex: avionic)

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- **Critical system:** correction is (very) important
- **Real-time:** hard temporal deadlines
- **Synchronous:**
  - Manipulate infinite streams of data
  - Computations can have different cadences of production
  - Rhythm of computations synchronized with a global clock

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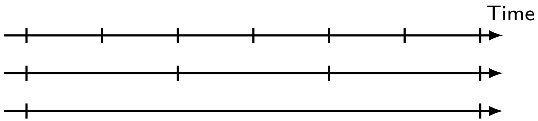

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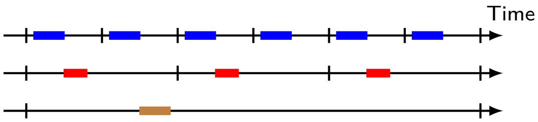

Harmonic periods
Computation: once per period

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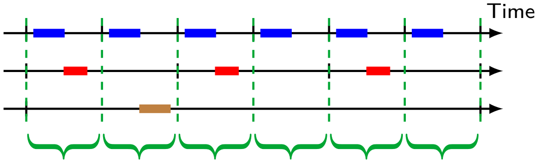

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1 iteration = 1 tick of fastest period

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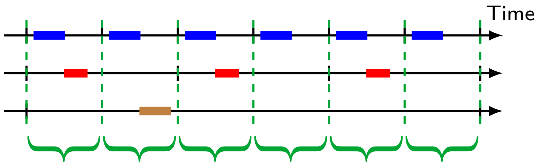

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- **Phase:** in which iteration should a computation be? Must respect timing constraints
  - Deadline at the end of a tick
  - (Additional) Latency constraints

Contributions

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2. Unknown phases, to be inferred by the compiler:
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   - Objective function: load-balancing
3. Under-specified operators: fuzzy dataflow
   - Possible to relax some dependency constraints

Background - Lustre programming language

- Equational language for synchronous program:
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x	0	1	1	2	...
y	4	-2	1	4	...
42	42	42	42	42	...
$x + y$	4	-1	2	6	...
42 `fby` y	42	4	-2	1	...

- `fby` operator: use the previous value (memory)

Background - Clocks

- **Clock**: $x :: clk$
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  - Arbitrary in general
  - Will consider only *periodic* clocks (ex: (T F T T))
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- Clocking rules: check that the clocks match
- **Sub/Over-sampling:** `when` / (`merge` + `current`)
  - `when`: filter the values of a (faster) stream
  - `merge`: combine 2 streams
  - `current`: repeat (faster) a value produced by a slower stream

x	$:: c$	0	1	1	2	...
$b = (TF)$	$:: c$	T	F	T	F	...
$z = x$ `when` b	$:: c$ `on` b	0	–	1	–	...
y	$:: c$ `on not` b	–	42	–	64	...
`merge` $b\ z\ y$	$:: c$	0	42	1	64	...
`current(b,0,z)`	$:: c$	0	0	1	1	...

1-synchronous clocks and temporal operators

- **1-synchronous clock** : for integration program
  - Periodic + only one activation per period
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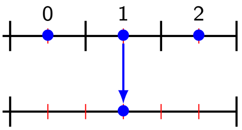

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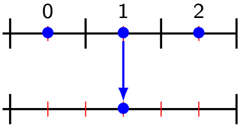

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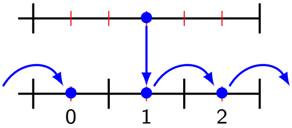

y = `current`([1,3], 42, x)

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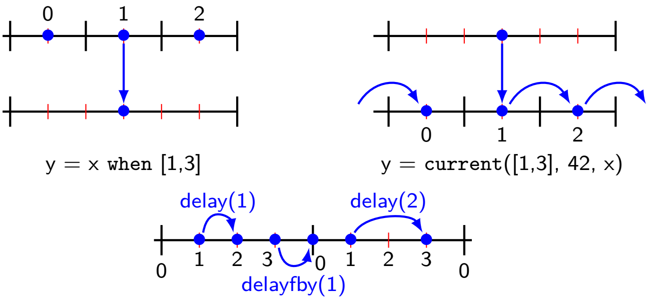
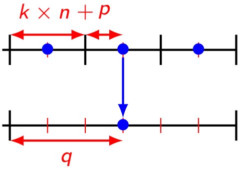


Clock analysis

- 1-synchronous operators: add a constant to the phase



y = x `when` [k,r]

where:

- p/q are phases, n/m periods
- k is the number of the sampled occurrence

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- **Clock analysis:** now manipulates symbolic phases p.
- Gather affine constraints from it:
  - Boundary condition on phases: $0 \leq p < n$
  - Dependence constraints from buffer: $p_1 + C \leq p_2$
  - Unification constraints from ops/equation: $p_1 = p_2 + C$
  - (Additional) latency constraints: $p_1 - p_2 \leq C$

Example

Example of program:

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model main(...)  returns (...)
var
  a ::  [..,1]; b ::  [..,2];
  c ::  [..,6]; d ::  [..,2]; e ::  [..,1];
let
  b = buffer f1(a when [1,2]);
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$\Rightarrow$ Provide this property to relax the scheduling problem

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- Add constraints on the d_i to avoid causality loops.

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\begin{aligned}
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- Load balancing constraint (minimize W_{max})

$$(\forall k \text{ phase}) \quad \sum_T (\delta_{k \ mod \ per(T),T} \times W_T) \leq W_{max}$$

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- Redundancy of encoding of phase value (binary + integer)

  $\Rightarrow$ Alternative formulation with binary only

Experimental results

- Two real-world industrial use cases:
  - **UC1:** (control command - Airbus) 5124 nodes, 32k variables 4 harmonic periods (10/20/40/120 ms)
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ILP problem		# of lines	# of vars	Solving time
UC1	No cost function	28742	5124	0.05 s
	Load balancing integral	79128	45249	17.75 s
	Load balancing binary	200746	40125	9.05 s
UC2	No cost function	3908	143	0.01 s
	Load balancing integral	5106	1039	0.03 s
	Load balancing binary	17689	897	0.05 s

- Solutions found are $\leq 0.01\%$ above rational optimal.

Injecting the solution in the program

- Once we found a solution, reinject it in the program
- With decision variable values: `op?` $\Rightarrow$ `op`
- With phase values:
  - `buffer` $\Rightarrow$ `delay(d)`
  - `[..,n]` $\Rightarrow$ `[k,n]`

$\Rightarrow$ After substitution, fully specified 1-synchronous program

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- Takeaway message: Semantic properties can be useful

  $\Rightarrow$ Check how an application is specified/used in practice

Thank you for listening. . .

. . . Do you have any questions?