Splitting Polyhedra to Generate More Efficient Code

Efficient Code Generation in the Polyhedral Model is Harder Than We Thought

January 23rd 2017, IMPACT'17, HiPEAC, Stockholm, Sweden

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Introduction to the $\mathbb{Z}$-Polyhedral Model

The polyhedral model — also known as the polytope model — is a mathematical abstraction that can be used to represent, manipulate and optimize loop nests.

Definition

A parametric polyhedron P_p with n parameters is a set of d-dimensional vectors x:

$$P_p = \left\{ x \in \mathbb{K}^d \mid Ax \geq Bp + c \right\}$$

where $A \in \mathbb{K}^{m \times d}$, $B \in \mathbb{K}^{m \times n}$ and $c \in \mathbb{K}^m$ so that for each fixed value p_0, P_{p_0} defines a polyhedron in $\mathbb{K}^d$.

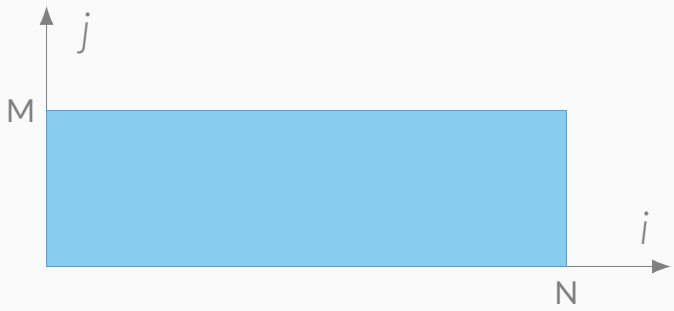
Introduction to the $\mathbb{Z}$-Polyhedral Model

```
1  for (i = 0; i <= N; ++i)
2    for (j = 0; j <= M; ++j)
3      S(i, j);
```

(a) Loop nest

$$\mathcal{S}(i,j) = \left\{ (i,j) \in \mathbb{Z}^2 \middle| \begin{array}{l} 0 \leq i \leq N \\ 0 \leq j \leq M \end{array} \right\}$$

(b) Constraints



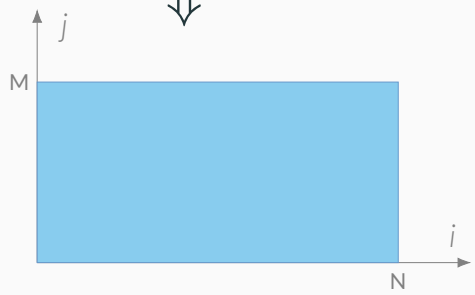
(c) 2D representation

Figure 1: Various representations of a loop nest in the polyhedral model

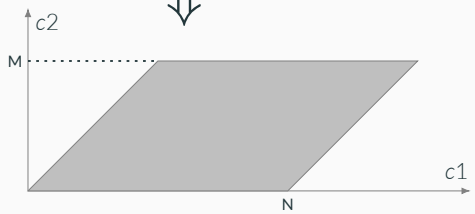
Overview of the Polyhedral Model Framework

```
1    for (i = 0; i <= N; ++i)
2      for (j = 0; j <= M; ++j)
3        S(i, j);
```

1 Raising



2 Transformation



3 Code generation

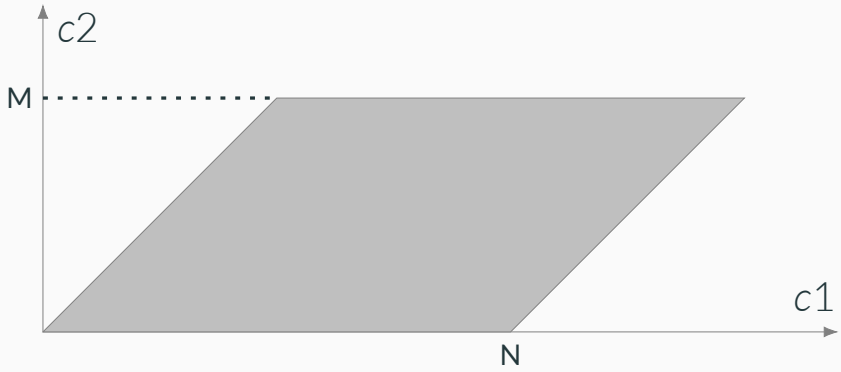
```
1    for (c1 = 0; c1 <= N+M; c1++)
2      for (c2 = max(0, c1-N); c2<= min(M, c1); c2++)
3        S1(c1, c2);
```

A Simple Polyhedron

Let's consider the following polyhedron:

$$\mathcal{P}(N, M) = \left\{ (c1, c2) \in \mathbb{Z}^2 \;\middle|\; \begin{array}{l} 0 \leq c1 \leq N + M \\ 0 \leq c2 \\ c1 - N \leq c2 \\ c2 \leq c1 \\ c2 \leq M \end{array} \right\}$$

(a) Constraints

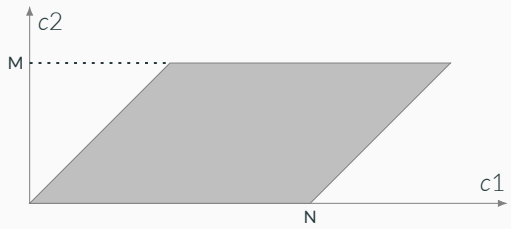


(b) 2D representation

Figure 2: A simple polyhedron

What code would you generate?

Code Generation is Easy!



Most tools would generate this:

```
1  for (c1 = 0; c1 <= N+M; c1++)
2    for (c2 = max(0, c1-N); c2<= min(M, c1); c2++)
3      S1(c1, c2);
```

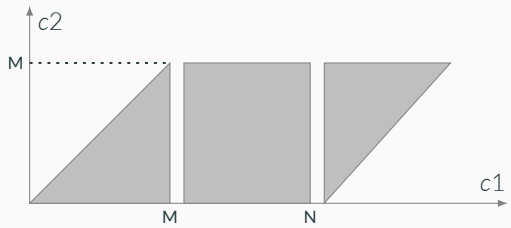
Listing 1: Code for scanning the polyhedron $\mathcal{P}$ from Figure 2

...and most people would be happy.

Are you?

Is Efficient Code Generation Easy?

What about this:




```
1  for (c1 = 0; c1 <= M; c1++)
2    for (c2 = 0; c2 <= c1; c2++)
3      S1(c1, c2);
4  for (c1 = M+1; c1 <= N-1; c1++)
5    for (c2 = 0; c2 <= M; c2++)
6      S1(c1, c2);
7  for (c1 = N; c1<= N+M; c1++)
8    for (c2 = c1-N; c2<= M; c2++)
9      S1(c1, c2);
```

Listing 2: Another way to scan the polyhedron $\mathcal{P}$ from Figure 2

Quick Comparison

```
1   for (c1 = 0; c1 <= N+M; c1++)
2     for (c2 = max(0, c1-N); c2<= min(M, c1); c2++)
3       S1(c1, c2);
```

- high control overhead
- small code size
- fast code generation

```
1   for (c1 = 0; c1 <= M; c1++)
2     for (c2 = 0; c2 <= c1; c2++)
3       S1(c1, c2);
4   for (c1 = M+1; c1 <= N-1; c1++)
5     for (c2 = 0; c2 <= M; c2++)
6       S1(c1, c2);
7   for (c1 = N; c1<= N+M; c1++)
8     for (c2 = c1-N; c2<= M; c2++)
9       S1(c1, c2);
```

- low control overhead
- increased code size
- slow code generation
- speedups reach 1.32 with gcc, 1.67 with icc and 4.63 with clang

Goals

Our goals:

- eliminate min and max computations in order to reduce control overhead
- help the compiler:
  - polyhedra splitting can ease vectorization and branch prediction
  - tiled iteration domains can benefit from polyhedra splitting
  - different splitting strategies impact the overall quality of generated code, code size and generation time

Outline

Introduction

Splitting polyhedra

- Prerequisites
- Splitting Polyhedra in QRW

Experimental results

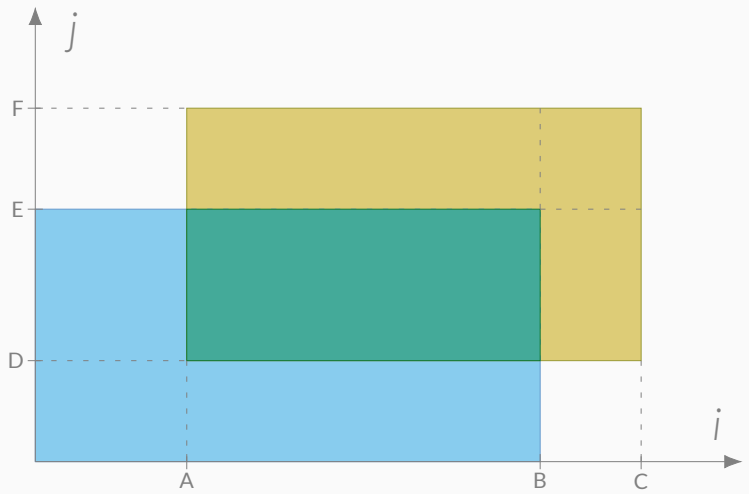
Conclusion

Reminder on CLooG

- CLooG: Chunky Loop Generator
- Generates code for scanning $\mathbb{Z}$-polyhedra
- Extended version of the Quilleré, Rajopadhye, and Wilde algorithm [2]

CLooG's extended QRW Algorithm

1. Intersect the polyhedra with the context
2. Project the polyhedra onto the outermost dimension
3. Separate the projections into a disjunct list of polyhedra
4. (Compute lexicographic order and strides)
5. Recurse on the inner dimensions



$$Context : \left\{ \begin{array}{l} 0 \leq A < B < C \\ 0 \leq D < E < F \end{array} \right\}$$

$$\mathcal{S}_1(B, E) = \left\{ (i, j) \in \mathbb{Z}^2 \;\middle|\; \begin{array}{l} 0 \leq i \leq B \\ 0 \leq j \leq E \end{array} \right\}$$

$$\mathcal{S}_2(A, C, D, F) = \left\{ (i, j) \in \mathbb{Z}^2 \;\middle|\; \begin{array}{l} A \leq i \leq C \\ D \leq j \leq F \end{array} \right\}$$

Figure 5: Intersection of the polyhedra with the context

CLooG's extended QRW Algorithm

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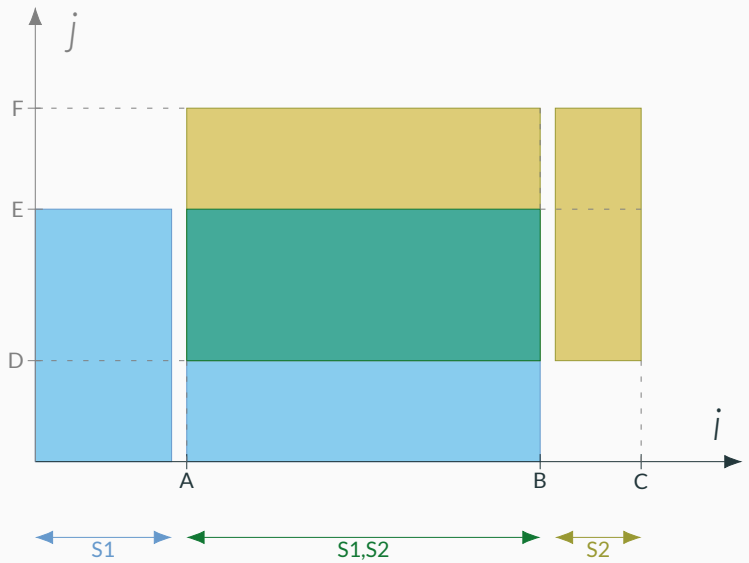
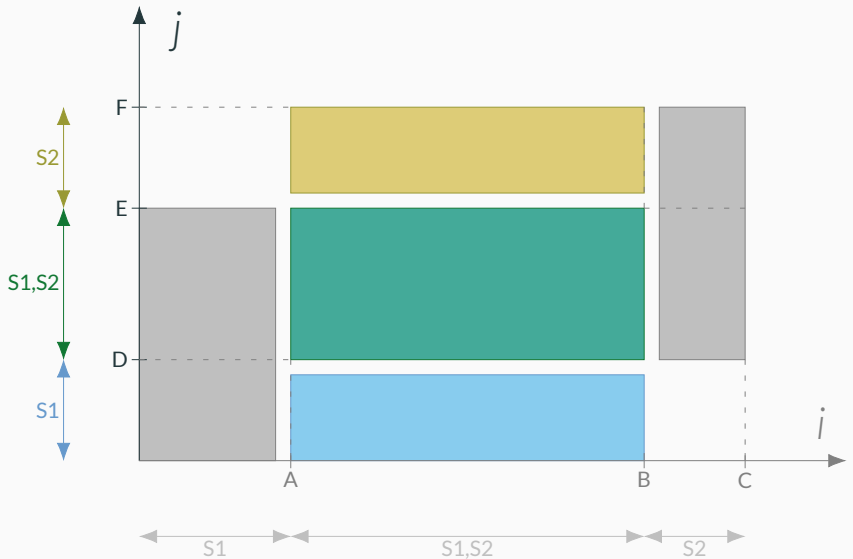



```
1   for (i = 0; i <= A-1; ++i)
2     /* S1 */
3   for (i = A; i <= B; ++i) {
4     /* S1 and S2 */
5   }
6   for (i = B+1; i <= C; ++i)
7     /* S2 */
```

Figure 6: Projection and separation on the first dimension

CLooG's extended QRW Algorithm

1. Intersect the polyhedra with the context
2. Project the polyhedra onto the outermost dimension
3. Separate the projections into a disjunct list of polyhedra
4. (Compute lexicographic order and strides)
5. Recurse on the inner dimensions




```
1   for (i = 0; i <= A-1; ++i)
2     for (j = 0; j <= E; ++j)
3       S1(i, j);
4   for (i = A; i <= B; ++i) {
5     for (j = 0; j <= D-1; ++j)
6       S1(i, j);
7     for (j = D; j <= E; ++j) {
8       S1(i, j);
9       S2(i, j);
10    }
11    for (j = E+1; j <= F; ++j)
12      S2(i, j);
13  }
14  for (i = B+1; i <= C; ++i)
15    for (j = D; j <= F; ++j)
16      S2(i, j);
```

Figure 7: Recursion on the second dimension

Chamber Decomposition

- Chamber : a part of the parameter domain in which all vertices coordinates are affine functions of parameters
- Assuming the number of parameters of a parameterized polyhedron is *m*, the validity domains are obtained by computing the *m*-faces of the polyhedron using Loechner and Wilde's algorithm [1]
- The PolyLib allows to compute the validity domains of parameterized polyhedra

Splitting Polyhedra in QRW

Introduction

Splitting polyhedra

Prerequisites

Splitting Polyhedra in QRW

Experimental results

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Splitting polyhedra in CLooG's extended QRW

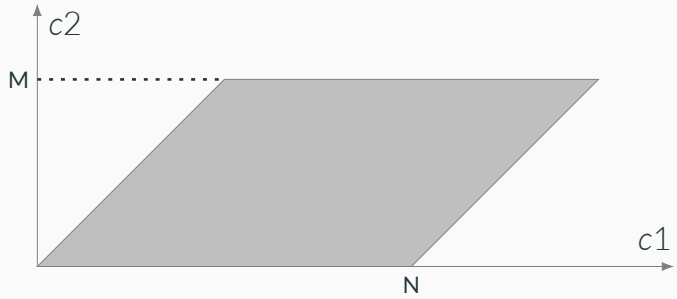
- After the separation step of CLooG’s algorithm
- Relies on chamber decomposition
- Previous parameters, outer loop indices and the current loop index must be considered as parameters

Example i

1. Let's consider the polyhedron from Figure 2 again:

$$\mathcal{P}(N,M) = \left\{ (c1,c2) \in \mathbb{Z}^2 \;\middle|\; \begin{array}{l} 0 \leq c1 \leq N+M \\ 0 \leq c2 \\ c1 - N \leq c2 \\ c2 \leq c1 \\ c2 \leq M \end{array} \right\}$$

(a) Constraints



(b) 2D representation

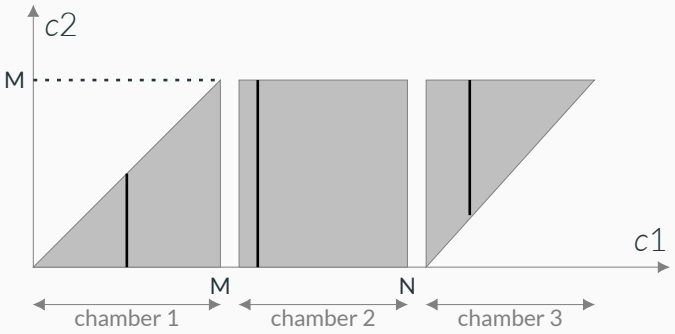
$$\mathcal{P}_{c1}(N,M,c1) = \left\{ (c2) \in \mathbb{Z} \;\middle|\; \begin{array}{l} 0 \leq c2 \\ c1 - N \leq c2 \\ c2 \leq c1 \\ c2 \leq M \end{array} \right\}$$

(c) Considering c1 as a parameter

Example ii

2. Computing the chambers of the parametric polyhedron $\mathcal{P}_{c1}$:

$$
\begin{aligned}
\mathcal{C}_1 &= \{(N, M, c1) \in \mathbb{Z}^3 | 0 \leq c1 \leq M\} \\
\mathcal{C}_2 &= \{(N, M, c1) \in \mathbb{Z}^3 | M + 1 \leq c1 \leq N - 1\} \\
\mathcal{C}_3 &= \{(N, M, c1) \in \mathbb{Z}^3 | N \leq c1 \leq N + M\}
\end{aligned}
$$



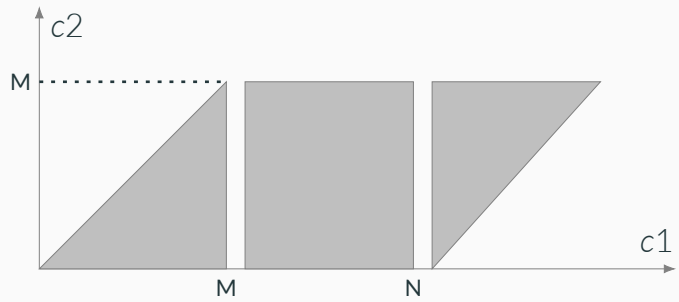
Example iii

3. Revert c1 to an iterator:

$$\mathcal{P}_1(N,M) = \left\{ (c1,c2) \in \mathbb{Z}^2 \,\middle|\, \begin{array}{l} 0 \le c1 \le M \\ 0 \le c2 \le c1 \end{array} \right\}$$

$$\mathcal{P}_2(N,M) = \left\{ (c1,c2) \in \mathbb{Z}^2 \,\middle|\, \begin{array}{l} M+1 \le c1 \le N-1 \\ 0 \le c2 \le M \end{array} \right\}$$

$$\mathcal{P}_3(N,M) = \left\{ (c1,c2) \in \mathbb{Z}^2 \,\middle|\, \begin{array}{l} N \le c1 \le N+M \\ c1-N \le c2 \le M \end{array} \right\}$$



Experimental Results

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Experimental Tools

Compring the results of the mainline CLooG and our PolyLib-enhanced version:

- CLooG's test suite and PolyBench 4.1
- Code scheduled with PLUTO 0.11.4 (option `--tile`)
- Code compiled using `gcc 6.2.1`, `clang 3.8.1` and `icc 17.0.0` with options `-O0` and `-O3 -march=native`
- Tested on a 2.40GHz Intel Xeon E5-2620v3, running linux 4.8.

Polybenches `-O0`

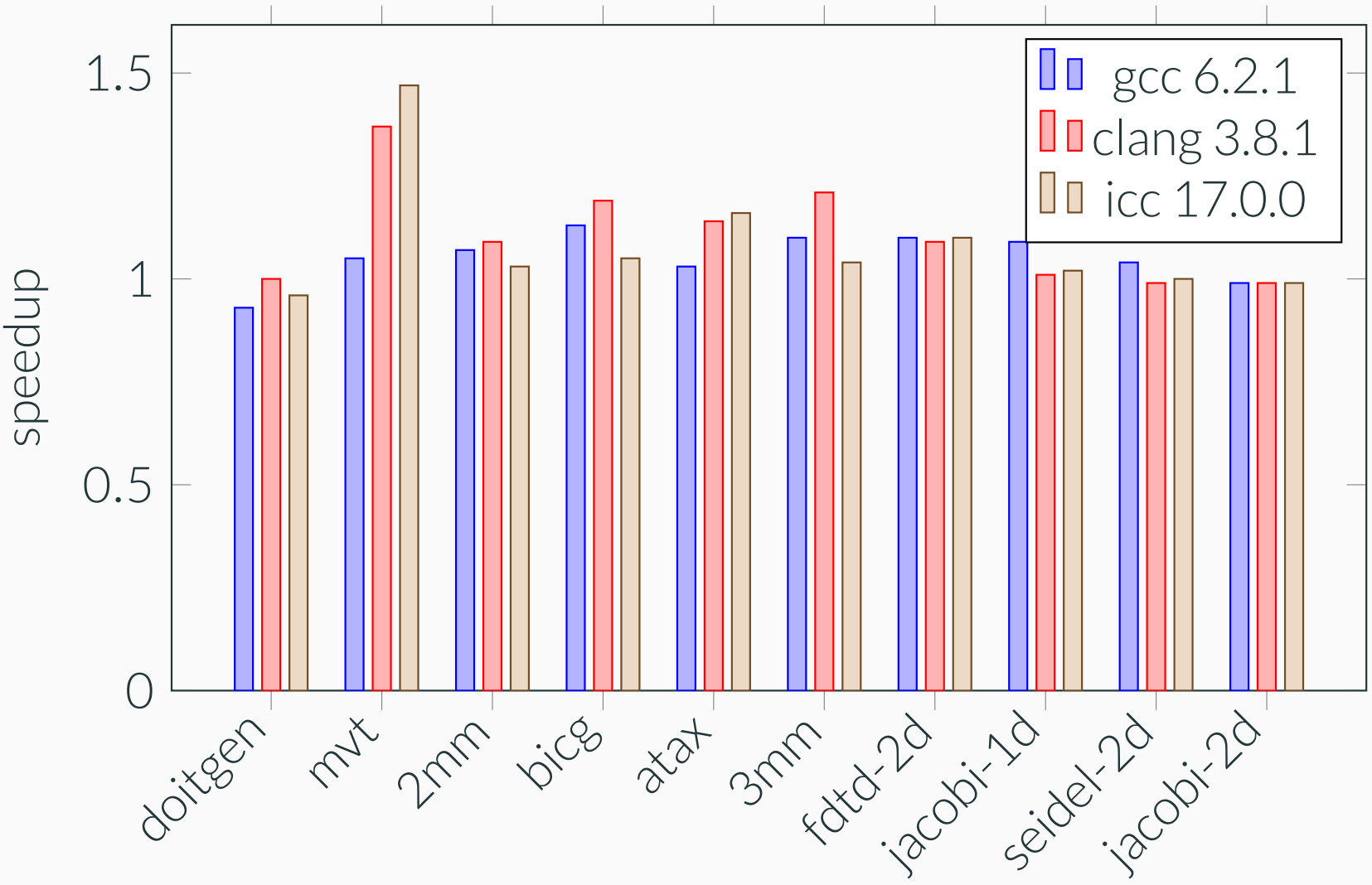

Figure 11: Speedups on the PolyBench test suite with `-O0`

Polybenches `-O3 -march=native`

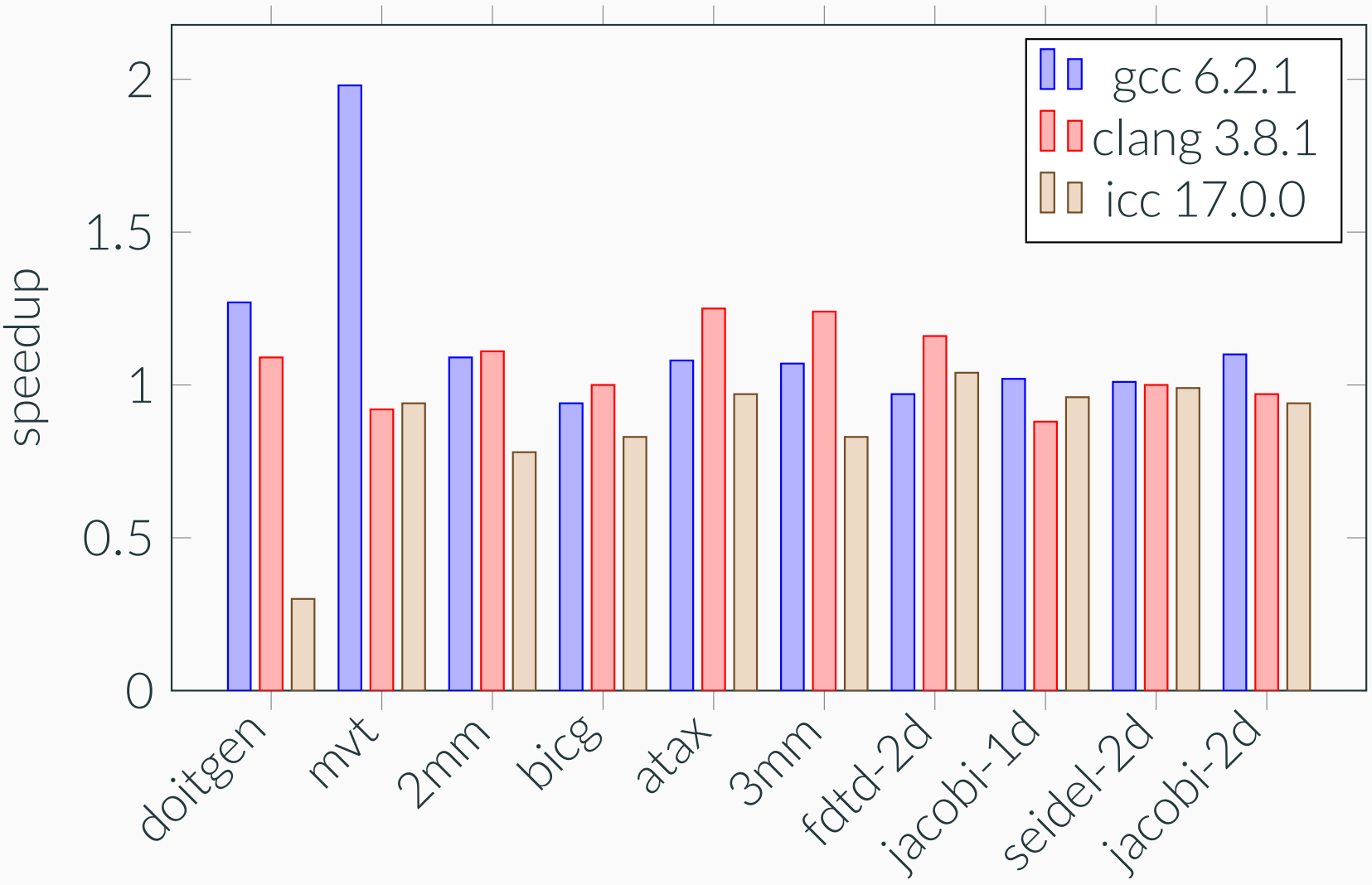

Figure 12: Speedups on the PolyBench tests with `-O3 -march=native`

Speedups on CLooG's Test Suite

	speedup > 1			speedup < 1			geomean
compiler	percentage	maximum	geomean	percentage	minimum	geomean	speedup
gcc -O0	59	1.34	1.07	41	0.81	0.96	1.02
icc -O0	66.7	1.21	1.03	33.3	0.97	0.99	1.01
clang -O0	82.5	1.96	1.19	17.5	0.5	0.9	1.12
gcc -O3	51.2	1.07	1.03	48.8	0.9	0.97	1.00
icc -O3	61.5	1.11	1.02	38.5	0.89	0.98	1.00
clang -O3	63.9	1.17	1.09	36.1	0.66	0.95	1.03

Figure 13: Overview of the speedups for the CLooG tests

Price to pay

- Slower CLooG code generation
  - 5 times slower for CLooG's test suite
  - 10 times slower for the PolyBench
- Increased code size (geometric mean growth of 257%)

Conclusion

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Conclusion

- A new method for splitting polyhedra in order to reduce control overhead
- Splitting polyhedra *may* improve the efficiency of the generated code
- Slower code generation and increased code size
- Future work: determining when such splitting should or should not be performed
- Efficient code generation is harder than we thought!

Thank you!

Références i

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Parameterized Polyhedra and Their Vertices.
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Generation of efficient nested loops from polyhedra.
International Journal of Parallel Programming, 28(5):469–498, 2000.