On Demand Parametric Array Dataflow Analysis

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Outline

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Motivation

- Dataflow analysis determines for read access in a statement instance, the statement instance that wrote the value being read
- Many uses in polyhedral analysis/compilation
  - array expansion
  - scheduling
  - equivalence checking
  - optimizing computation/communication overlap in MPI programs
  - derivation of process networks
  - ...
- Standard dataflow analysis (Feautrier) requires static affine input programs
- Extensions are needed for programs with dynamic/non-affine constructs

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- Standard dataflow analysis (Feautrier) requires static affine input programs
- Extensions are needed for programs with dynamic/non-affine constructs

Our Motivation: Derivation of Process Networks

- Main purpose: extract task level parallelism from dataflow graph

$$\begin{array}{rcl} \text{statement} & \rightarrow & \text{process} \\ \text{flow dependence} & \rightarrow & \text{communication channel} \end{array}$$

  $\Rightarrow$ requires dataflow analysis
- Processes are mapped to parallel hardware (e.g., FPGA)

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Example:

```
for (i = 0; i < n; ++i) {
        a = f();
        g(a);
}
```

Dynamic Process Networks

```
int state = 0;
for (i = 0; i <= 10; i++) {
  sample = radioFrontend();
  if (state == 0) {
    state = detect(sample);
  } else {
    state = decode(sample, &value0);
    value1 = processSample0(value0);
    processSample1(value1);
  }
}
```

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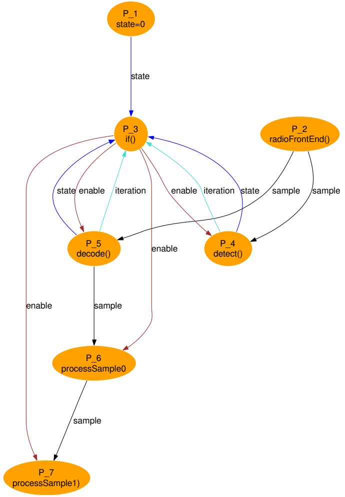

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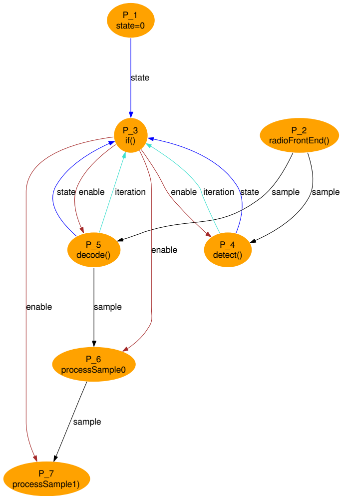

- additional control channels
- determine operation of data channels
- dataflow analysis needs to remain exact, but may depend on run-time information

Outline

Standard Array Dataflow Analysis

Given a read from an array element, what was the last write to the same array element before the read?

Simple case: array written through a single reference

```
for (i = 0; i < N; ++i)
    for (j = 0; j < N - i; ++j)
F:      a[i+j] = f(a[i+j]);
for (i = 0; i < N; ++i)
W:  Write(a[i]);
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Access relations:

```
A1:=[N]->{F[i,j]->a[i+j]:0<=i<N and 0<=j<N-i};
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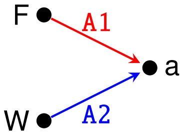

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Map to all writes: `R := A2 . (A1^-1);`

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[N] -> { W[i] -> F[i',i-i'] : 0 <= i,i'< N and i'<= i }
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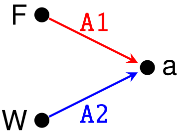

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Last write: `lexmax R; # [N] -> { W[i] -> F[i,0] : 0 <= i < N }`

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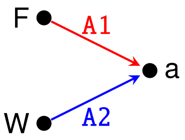

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Last write: `lexmax R; # [N] -> { W[i] -> F[i,0] : 0 <= i < N }`

In general: impose lexicographical order on shared iterators

Standard Array Dataflow Analysis

Multiple Potential Sources

- Dataflow is typically performed per read access ("sink") C
- Corresponding writes ("potential sources") P are considered in turn
- Map to all potential source iterations: $D^{\mathrm{mem}}_{C,P} = (A_P^{-1} \circ A_C) \cap B_C^P$ ("memory based dependences"; B_C^P: P executed before C)
- Source may already be known for some sink iterations
  $\Rightarrow$ compute *partial* lexicographical maximum

$$(U', D) = \underset{U}{\operatorname{lexmax}} M$$

U: sink iterations for which no source has been found
M: part of memory based dependences for particular potential source
$U' = U \setminus \operatorname{dom} M$
$M' = \operatorname{lexmax}(M \cap (U \to \operatorname{ran} M))$

Note: here, dependence relations map sink iterations to source iterations

Fuzzy Array Dataflow Analysis

- Introduces parameters for each lexmax involving dynamic behavior
- Parameters represent dynamic solution of lexmax operation
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Main problem for deriving process networks:
Introduces too many parameters
$\Rightarrow$ too many control channels

On Demand Parametric Array Dataflow Analysis

Similar to FADA:

- Exact, possibly parametric, dataflow
- Introduces parameters to represent dynamic behavior

But:

+ Parameters have a different meaning
+ Effect analyzed before parameters are introduced
+ All computations are performed directly on affine sets and maps
– Currently only supports dynamic conditions

Outline

Representing Generic Dynamic Conditions

```
   while (1) {
       sample = radioFrontend();
       if (t(state)) {
D:         state = detect(sample);
       } else { /* ... */ }
   }
```

Representing Generic Dynamic Conditions

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Dynamic condition (`t(state)`) represented by *filter*

- Filter access relation(s):
  access to (virtual) array representing condition

$$\{\, D(i) \to (S_0(i) \to t_0(i)) \,\}$$

- Filter value relation:
  values of filter array elements for which statement is executed

$$\{\, \mathtt{D}(i) \to (1) \mid i \geq 0 \,\}$$

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statement reading from filter array

- Filter value relation:
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Representing Locally Static Affine Conditions

```
N1: n = f();
    for (int k = 0; k < 100; ++k) {
M:      m = g();
        for (int i = 0; i < m; ++i)
            for (int j = 0; j < n; ++j)
A:              a[j][i] = g();
N2:     n = f();
    }
```

Values of `m` and `n` not changed inside `i` and `j` loops

$\Rightarrow$ locally static affine loop conditions

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- Filter access relations:
  $\{\, \mathtt{A}(k,i,j) \to (\mathtt{M}(k) \to \mathtt{m}()) \,\}$
  $\{\, \mathtt{A}(0,i,j) \to (\mathtt{N1}() \to \mathtt{n}()) \,\} \cup \{\, \mathtt{A}(k,i,j) \to (\mathtt{N2}(k-1) \to \mathtt{n}()) \mid k \geq 1 \,\}$
- Filter value relation:
  $\{\, \mathtt{A}(k,i,j) \to (m,n) \mid 0 \leq k \leq 99 \wedge 0 \leq i < m \wedge 0 \leq j < n \,\}$

Note: filter access relations exploit (static) dataflow analysis on `m` and `n`

Outline

Overview

- Dataflow analysis performed for each read access (sink) separately
- Potential sources considered from closest to furthest
  - number of shared loop iterators ℓ
  - textual order
- For each lexmax operation
  - is it possible for potential source not to execute when sink is executed? (based on filters)
  - if so, parametrize lexmax problem

Parametrization

```
   state = 0;
   while (1) {
       sample = radioFrontend();
       if (t(state)) {
D:         state = detect(sample);
       } else {
C:         decode(sample, &state, &value0);
           value1 = processSample0(value0);
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sink *C*

potential source *P*

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sink C

potential source P

Memory based dependences: $D^{\text{mem}}_{C,P} = \{\, \mathtt{S_0}(i) \to \mathtt{D}(i') \mid 0 \le i' < i \,\}$

At $\ell = 1$: $M = D^{\text{mem}}_{C,P} \cap \{\, \mathtt{S_0}(i) \to \mathtt{D}(i) \,\} = \emptyset$

At $\ell = 0$: $M = \{\, \mathtt{S_0}(i) \to \mathtt{D}(i') \mid 0 \le i' < i \,\}$

Potential source $\mathtt{D}(i')$ may not have executed even if sink $\mathtt{S_0}(i)$ is executed

$\Rightarrow$ parametrization required

Parameter Representation

Original:

$$M = \{\, \mathtt{S_0}(i) \to \mathtt{D}(i') \mid 0 \le i' < i \,\}$$

After parameter introduction:

$$M' = \left\{ \mathtt{S_0}(i) \to \mathtt{D}(\lambda_C^P(i)) \mid 0 \le \lambda_C^P(i) < i \wedge \beta_C^P(i) = 1 \right\}$$

$\Rightarrow \operatorname{lexmax} M' = M'$

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Meaning of the parameters:

- $\lambda_C^P(\mathbf{k})$: last executed iteration of $D_{C,P}^{\mathrm{mem}}(\mathbf{k})$
- $\beta_C^P(\mathbf{k})$: any iteration of $D_{C,P}^{\mathrm{mem}}(\mathbf{k})$ is executed

Note: FADA introduces separate set of parameters for each lexmax
Note: $\lambda_C^P(\mathbf{k})$ and $\beta_C^P(\mathbf{k})$ depend on $\mathbf{k}$, but dependence can be kept implicit
$\Rightarrow \lambda_C^P$ and β_C^P

Introducing as few Parameters as possible

In principle, the number of elements in λ is equal to the number of iterators
However, in many cases, we can avoid introducing some of those elements

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```
for (i = 0; i < 100; ++i)
        if (t())
                for (j = 0; j < 100; ++j)
A:                      a = t();
B: b = a;
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$$M = \{ B() \to A(i,j) \mid 0 \le i,j < 100 \}$$

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$$M' = \{ B() \to A(\lambda_0,j) \mid 0 \le \lambda_0, j < 100 \wedge \beta = 1 \}$$

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$$\mathrm{lexmax}\, M' = \{ B() \to A(\lambda_0, 99) \mid 0 \le \lambda_0 < 100 \wedge \beta = 1 \}$$

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- dimensions inside innermost condition that is not static affine
- dimensions that can only attain a single value (for a given value of $\mathbf{k}$)

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Dimensions that can only attain a single value

```
for (int k = 0; k < 100; ++k) {
N:        N = f();
M:        M = g();
          for (int i = 0; i < N; ++i)
                    for (int j = 0; j < M; ++j)
A:                            a[i][j] = i + j;
          for (int i = 0; i < N; ++i)
                    for (int j = 0; j < M; ++j)
H:                            h(i, j, a[i][j]);
}
```

$$D^{\mathrm{mem}}_{\mathrm{H,A}} = \{\, \mathtt{H}(k,i,j) \to \mathtt{A}(k',i,j) \mid k' \le k \,\}$$

$$\lambda_1(k,i,j) = i$$

$$\lambda_2(k,i,j) = j$$

$\Rightarrow$ no need to introduce λ_1 and λ_2

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Dimensions before ℓ

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At $\ell = 1$:

$$M = \{\, \mathtt{H}(k, i, j) \to \mathtt{A}(k, i, j) \,\}$$

$\Rightarrow$ no need to introduce λ_0 (yet) at $\ell = 1$

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Note: all sinks are accounted for at $\ell = 1$

$\Rightarrow$ no need to consider $\ell = 0$ and λ_0 not needed at all

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  $\Rightarrow$ replace β by σ: the number of implicitly equal shared iterators

$$
\begin{aligned}
\beta = 1 \quad &\rightarrow \quad \sigma \geq \ell \\
\beta = 0 \quad &\rightarrow \quad \sigma < \ell
\end{aligned}
$$

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  $\Rightarrow$ replace β by σ: the number of implicitly equal shared iterators

$$\begin{array}{lcl} \beta = 1 & \rightarrow & \sigma \geq \ell \\ \beta = 0 & \rightarrow & \sigma < \ell \end{array}$$

  - when moving to $\ell - 1$
    - ★ introduce additional parameter $\lambda_{\ell-1}$ (if needed)
    - ★ make implicit equality explicit
  - at the end of the dataflow analysis

$$\begin{array}{lcl} \sigma \geq \ell_{\leq} & \rightarrow & \beta = 1 \\ \sigma < \ell_{\leq} & \rightarrow & \beta = 0 \end{array}$$

    ($\ell_{\leq}$: smallest ℓ for which parametrization was applied)
    $\lambda(\mathbf{k})$ and $\beta(\mathbf{k})$ now refer to last execution of $\overline{D}(\mathbf{k})$
    ($\overline{D}$: result of projecting out parameters from final dataflow relation)

When to Introduce Parameters

- Sink C
- Potential source P
- Subset of sink iteration U
- Mapping to potential source iterations M

Computing

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$$(U', D) = \operatorname*{lexmax}_U M$$

1. No filter on source
   $\Rightarrow$ stop (no parametrization required)

When to Introduce Parameters

- Sink C
- Potential source P
- Subset of sink iteration U
- Mapping to potential source iterations M

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Filter on source contradicts F

$\ell = 1$

```
while (1) {
N:   n = f();
     a = g();
     if (n < 100)
H:         a = h();
     if (n > 200)
T:         t(a);
}
```

potential source

sink

- Potential source filter access relation

$$\left\{ \mathtt{H}(i) \to (\mathtt{N}(i) \to \mathtt{n}) \right\}$$

- Potential source filter value relation

$$\left\{ \mathtt{H}(i) \to (n) \mid i \geq 0 \wedge n < 100 \right\}$$

- Sink filter access relation

$$\left\{ \mathtt{T}(i) \to (\mathtt{N}(i) \to \mathtt{n}) \right\}$$

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Filter on source contradicts F

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same filter element

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while (1) {
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```

potential source

sink

contradiction

- Potential source filter access relation

$$\left\{ \mathtt{H}(i) \to \left(\boxed{\mathtt{N}(i) \to \mathtt{n}} \right) \right\}$$

- Potential source filter value relation

$$\left\{ \mathtt{H}(i) \to (n) \mid i \geq 0 \wedge \boxed{n < 100} \right\}$$

- Sink filter access relation

$$\left\{ \mathtt{T}(i) \to \left(\boxed{\mathtt{N}(i) \to \mathtt{n}} \right) \right\}$$

- Sink filter value relation

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4. Let F' be equal to F updated with information from other sources
5. Filter on source contradicts F'
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Filter on source contradicts F'

```
N:   n = f();
     if (n < 100)
H:       a = h();
     if (n < 200)
H2:      a = h2();
T:   t(a);
}
```

potential source

sink

$$\operatorname*{lexmax}_{U} M$$

$$M = \{\, \mathtt{T}() \to \mathtt{H}() \,\}$$

$$U = \left\{\, \mathtt{T}() \mid \sigma^{\mathrm{H2}} < 0 \,\right\}$$

Filter on source contradicts F'

```
N:   n = f();
     if (n < 100)
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     if (n < 200)
H2:        a = h2();
T:   t(a);
}
```

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sink

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H2 not executed

Filter on source contradicts F'

```
N:   n = f();
     if (n < 100)
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     if (n < 200)
H2:       a = h2();
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}
```

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H2 not executed

- Updated sink filter access relation

$$\left\{ \mathtt{T}(i) \to (\mathtt{N}(i) \to \mathtt{n}) \right\}$$

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3. Filter on source contradicts F
   $\Rightarrow$ replace M by empty relation and stop
4. Let F' be equal to F updated with information from other sources
5. Filter on source contradicts F'
   $\Rightarrow$ replace M by empty relation and stop
6. Filter on source implied by F
   $\Rightarrow$ stop (no parametrization required)

Filter on source implied by F

$$\ell = 1$$

```
while (1) {
N:  n = f();
    a = g();
    if (n < 200)
H:       a = h();
    if (n < 100)
T:       t(a);
}
```

potential source

sink

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7. Filter on source implied by F'
   $\Rightarrow$ parametrize D and stop

Filter on source implied by F'

```
N:   n = f();
     if (n < 200)
H:         a = h();
     if (n > 100)
H2:        a = h2();
T:   t(a);
}
```

potential source

sink

$$\operatorname*{lexmax}_{U} M$$

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- Updated sink filter access relation

$$\left\{\, \mathtt{T}(i) \to (\mathtt{N}(i) \to \mathtt{n}) \,\right\}$$

- Updated sink filter value relation

$$\left\{\, \mathtt{T}(i) \to (n) \mid i \geq 0 \land n \leq 100 \,\right\}$$

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6. Filter on source implied by F
   $\Rightarrow$ stop (no parametrization required)
7. Filter on source implied by F'
   $\Rightarrow$ parametrize D and stop
8. Parametrize M

Additional Constraints on Parameters

- Some source iterations are definitely executed
  $\Rightarrow \lambda$ no later than definitely executed iterations

Additional Constraints on Parameters

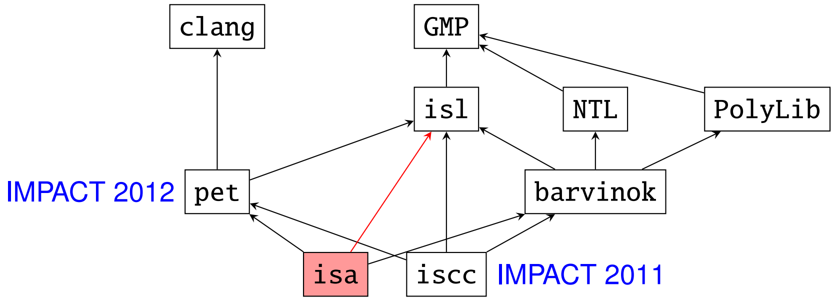
- Some source iterations are definitely executed
  $\Rightarrow \lambda$ no later than definitely executed iterations
- Eliminate (some) conflicts with other parameters

```
   state = 0;
   while (1) {
        sample = radioFrontend();
        if (t(state)) {
D:           state = detect(sample);
        } else {
C:           decode(sample, &state, &value0);
             value1 = processSample0(value0);
             processSample1(value1);
        }
   }
```

$\Rightarrow \lambda_0^{C}(i)$ and $\lambda_0^{D}(i)$ cannot both be smaller than $i - 1$

Outline

Interaction with Libraries



`isl`: manipulates parametric affine sets and relations

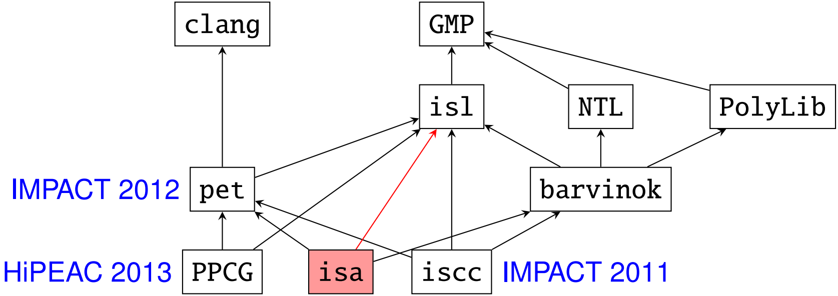
`barvinok`: counts elements in parametric affine sets and relations

`pet`: extracts polyhedral model from clang AST

`isa`: prototype tool set including

- derivation of process networks (with On Demand Parametric ADA)
- equivalence checker

Interaction with Libraries



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`PPCG`: Polyhedral Parallel Code Generator

Related Work

- Fuzzy Array Dataflow Analysis
  ⇒ only known publicly available implementation: `fadatool`
- Pugh et al. (1994) and Maslov (1995) produce approximate results
- Collard et al. (1999)
  - handle unstructured programs
  - only collect constraints
  - assume `Omega` can solve the constraints, but it cannot

Outline

Experimental Results

input	da			fadatool			fadatool -s		
	time	p	d	time	p	l	time	p	l
Example from paper	0.01s	0	5	0.01s	6	6	0.01s	6	6
Example from slides	0.01s	4	9	0.01s	6	16	incorrect		
fuzzy4	0.06s	3	9	0.02s	4	9	0.01s	0	9
for1	0.02s	2	3	0.01s	4	46	0.02s	2	3
for2	0.03s	2	3	0.09s	12	5k	0.04s	4	3
for3	0.04s	2	3	42s	24	1M	0.08s	6	3
for4	0.06s	2	3				0.16s	8	3
for5	0.08s	2	3				0.25s	10	3
for6	0.14s	2	3				0.42s	12	3
cascade_if1	0.02s	2	3	0.01s	2	4	0.01s	2	4
cascade_if2	0.02s	2	10	0.02s	4	52	0.02s	2	8
cascade_if3	0.03s	2	22	0.03	6	723	0.36s	3	16
cascade_if4	0.02s	2	10	0.17s	8	9k	1m	4	28
while1	0.01s	0	4	0.00s	1	4	0.01s	0	4
while2	0.03s	3	4	0.01s	5	6	incorrect		
if_var	0.03s	4	3	0.01s	2	8	0.01s	2	4
if_while	0.04s	2	14	0.01s	5	58	0.02s	4	58
if2	0.02s	2	2	0.46s	12	29k	0.04s	4	2

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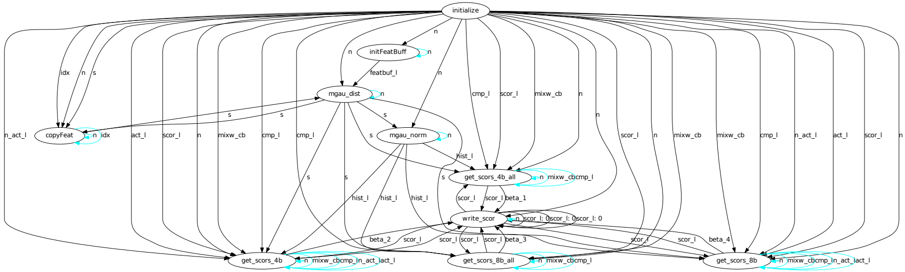
cascade_if4	0.02s	2	10	0.17s	8	9k	1m	4	28
while1	0.01s	0	4	0.00s	1	4	0.01s	0	4
while2	0.03s	3	4	0.01s	5	6	incorrect		
if_var	0.03s	4	3	0.01s	2	8	0.01s	2	4
if_while	0.04s	2	14	0.01s	5	58	0.02s	4	58
if2	0.02s	2	2	0.46s	12	29k	0.04s	4	2

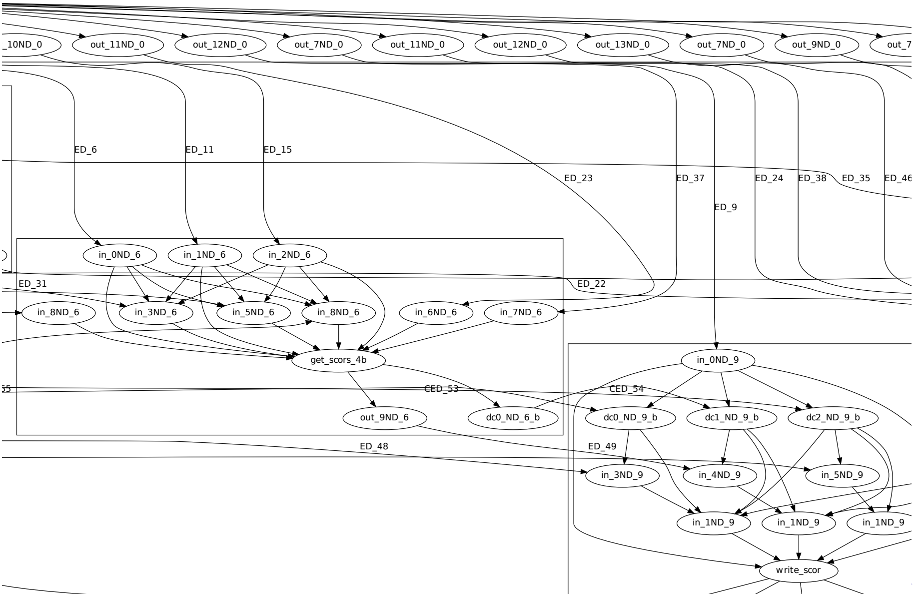
Larger Example — Input

```
for (j = 1; j <= frame; j++) {
  initialize(frame, n_act, &scor, &act, &ps, cmp,
             &s, &n, &idx, &mixw_cb, &cmp_l, &n_act_l, &act_l, &scor_l)
  for (i = 0; i < n; ++i) {
    initFeatBuff(i, &feat_buff, &featbuf_l);
    copyFeat(&s, frame, i, idx, &s);
    mgau_dist(&s, frame, i, &featbuf_l, &s);
    hist_l = mgau_norm(&s, frame, i);
    if (mixw_cb >= 1) {
      if (cmp_l >= 1)
        get_scors_4b_all(&s, i, hist_l, &scor_l, &scor_l);
      else
        get_scors_4b(&s, i, hist_l, n_act_l, &act_l, &scor_l, &scor_l);
    } else {
      if (cmp_l >= 1)
        get_scors_8b_all(&s, i, hist_l, &scor_l, &scor_l);
      else
        get_scors_8b(&s, i, hist_l, n_act_l, &act_l, &scor_l, &scor_l);
    }
    write_scor(&scor_l, &scor_l);
  }
}
```

Larger Example — Dataflow Graph



Larger Example — (Partial) Process Network



Outline

Conclusion

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- Dynamic behavior represented using “filters”
- Exact, possibly parametric, dataflow analysis
- Prototype implementation in `isa`
- Similar to FADA, but
  - Parameters have a different meaning
  - Effect analyzed before parameters are introduced
  - All computations are performed directly on affine sets and maps

Future work

- Tighter integration into `pet`